HyPerComp **I**ncompressible **M**HD solver for **A**rbitrary **G**eometry

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A summary of progress under two ongoing SBIR contracts:

Phase-I: Development of a canonical approach to liquid metal MHD computation and experiments

Phase-II Practical Simulations of two-phase MHD flows with wall effects

Outline

- Summary of tasks under the two SBIR projects with highlights
- Overview of applications under study of interest to the PFC community
- Future Plans

Most tasks ongoing / completed are shown in RED

Phase-I: Development of a canonical approach to liquid metal MHD computation and experiments

- Computationally parameterize criteria for a fully developed flow for a variety of channel flow situations: square channels with conducting walls, vs. Ha.
- Also, study the effect of magnetic field gradients (fringing fields) on flow development.
- Develop time accurate results database for 2-D circular cylinder, with and without conducting walls. Drag force as well as vortex frequency measurements are of interest.
- Driven cavity and broken dam benchmarks: Set up appropriate bench mark problems in MHD, equivalent to these standard test cases from conventional fluid dynamics
- Design a series of experiments for Phase-II to demonstrate canonical problems: There is already a test program at UCLA dedicated to fusion related studies. However, these must be modified to suite the needs of code validation.
- Initiate a discussion for interfacing with ongoing test programs to generate a MHD-test consortium. This may be across the ALPS, Plasma Chamber and other communities.

Code Improvements under Phase-I

1. Initiate turbulence modeling in HIMAG

    A general purpose advection-diffusion routine has been set up. This can be used for modeling processes such as heat conduction, k-e turbulence, etc.

2. Reduced dimensions for Fully developed 2D flow

HIMAG now has a “fully developed flow” option

<u>Required by the need to benchmark 3D vs 2D effects</u>

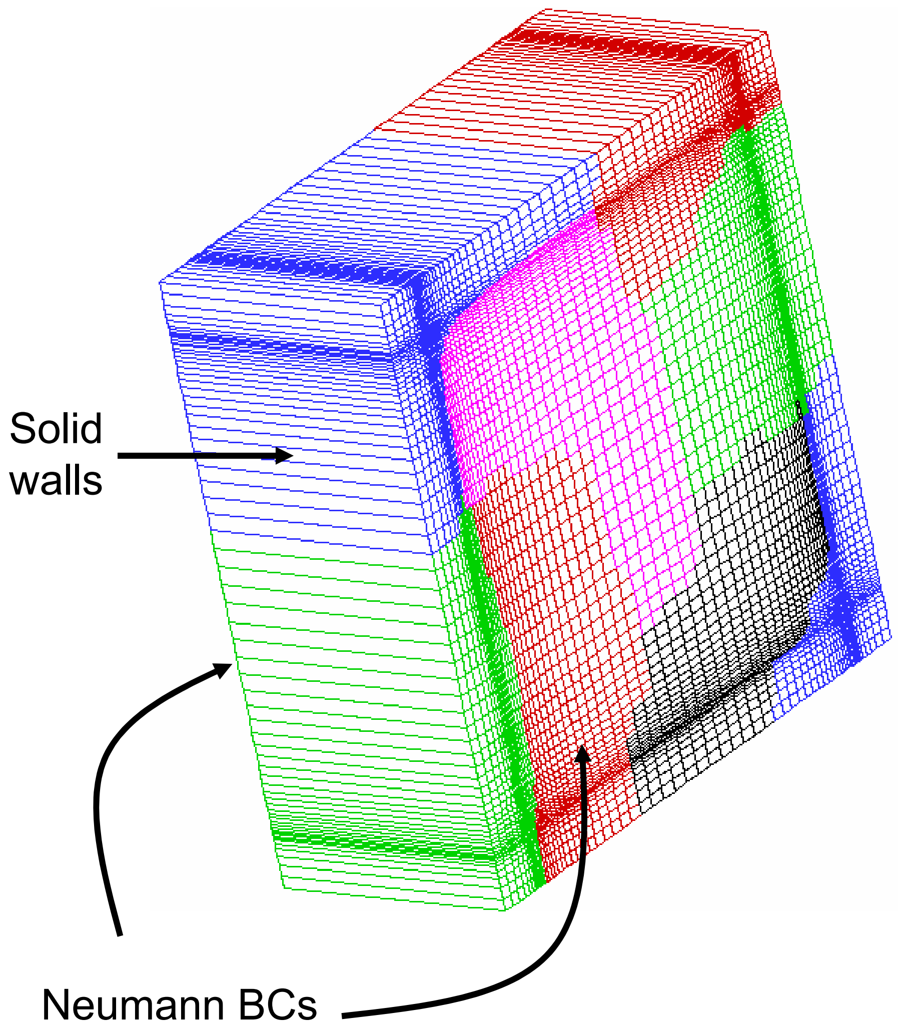
Single cell in the flow direction

Initial velocity can be set to zero and dp/dx = -c

Multiple conducting walls may be used

Parallelization is unaffected

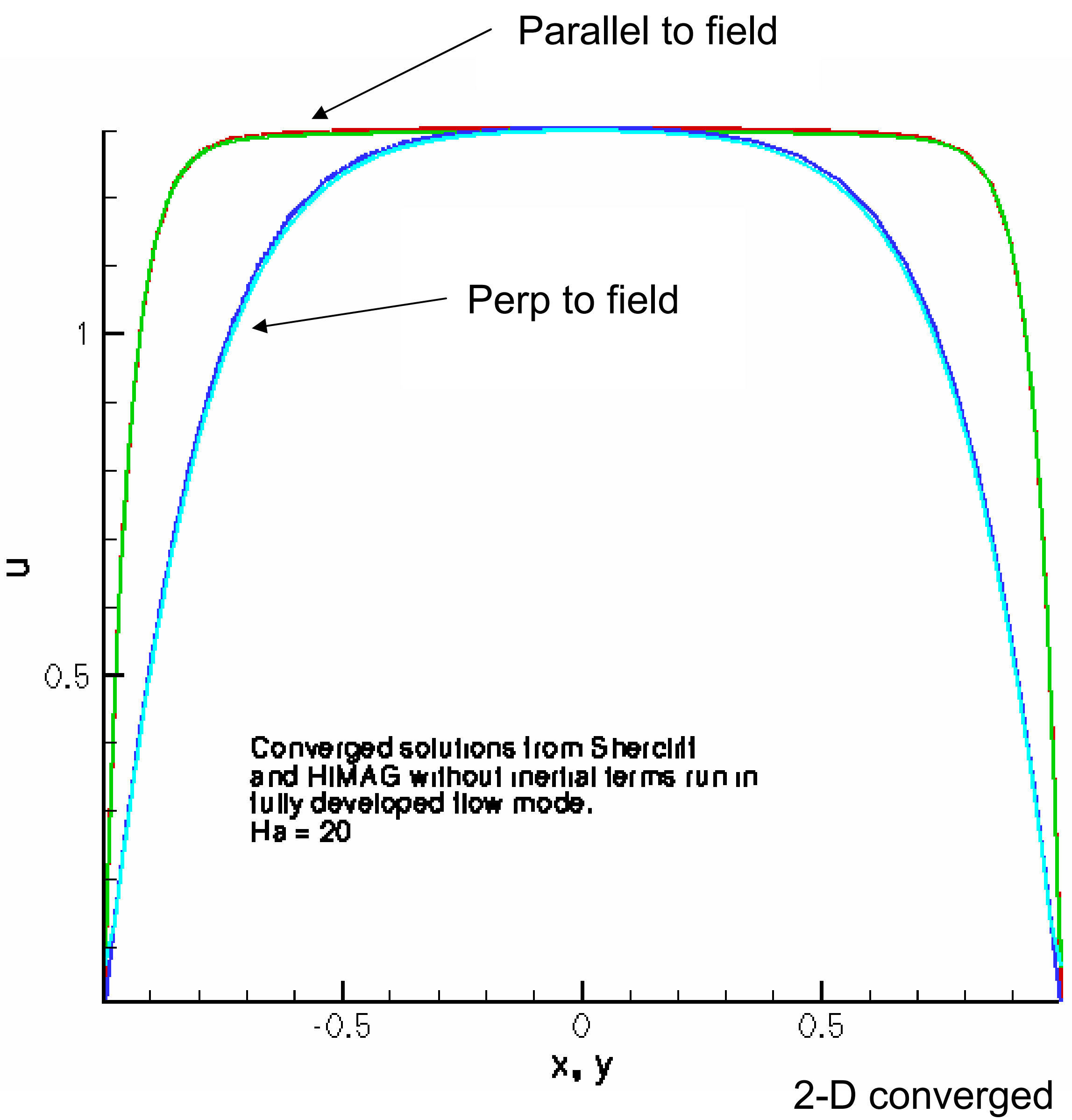
Code can be run with or without inertia terms



Looking back at closed channel cases with the fully developed version of HIMAG

There were concerns about the matching 2D analytic solutions to developing 3-D flows (a few Percent discrepancy)

Ongoing work benchmarks evolving vs. developed flow results and the effect of 3-D B-fields.



Phase-II: Practical Simulations of two-phase MHD flows with wall effects

The primary goals are:

(1) To enhance code capabilities in solid wall modeling (arbitrary conductivity, and including ferromagnetism)

(2) To enhance high Hartmann number capabilities, complete a formal validation for high Hartmann and Reynolds numbers for single and two phase flows

(3) To participate in code applications to engineering design cases in NSTX, DiMES, ITER

Tasks under Phase-II

1. Basic physics enhancements:
   - Induction equation (B)-formulation
   - Divergence control (both B and φ)
   - Thin conducting wall
   - Hartmann layer analytical approximations
   - Mixed Finite Volume-Boundary Element Methods for BC formulation on B
   - Cracks, contact resistance
   - Ferromagnetism

Tasks under Phase-II

2. Validation and demonstration
    - Closed channel flows – High B cases
    - Cracks in insulation
    - Jet and film flows
    - Co-ordinate with Phase-I

Tasks under Phase-II

3. Computational technology enhancement
   - Fast Poisson solvers: finish the CG technique for all variables
   - Code cleanup - restart
   - Parallel efficiency
   - Order of accuracy and stability

The **B**-formulation

The magnetic induction equation in the conservation form:

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times \left(\vec{V} \times \vec{B} \right) = -\vec{\nabla} \times \left(\frac{\vec{\nabla} \times \vec{B}}{\mu_0 \sigma} \right)$$

Integrating over a control volume, and using the Euler implicit scheme, we get:

$$\frac{B_i^{n+1} - B_i^n}{\Delta t} = \frac{1}{\Omega} \oint_{\partial\Omega} \left\{ \hat{n} \times \left(\vec{V} \times \vec{B} \right) \right\}_i^{n+1} ds - \frac{1}{\Omega} \oint_{\partial\Omega} \frac{1}{\mu_0 \sigma} \left\{ \hat{n} \times \left(\vec{\nabla} \times \vec{B} \right) \right\}_i^{n+1} ds$$

The finite volume expression is then written as:

$$B_i^{n+1} = B_i^n + \frac{\Delta t}{\Omega} \sum_{faces} \left\{ \hat{n} \times \left(\vec{V} \times \vec{B} \right) \right\}_i^{n+1} \Delta s - \frac{\Delta t}{\Omega} \sum_{faces} \frac{1}{\mu_0 \sigma} \left\{ \hat{n} \times \left(\vec{\nabla} \times \vec{B} \right) \right\}_i^{n+1} \Delta s$$

This equation is solved iteratively to convergence.

In the fully developed flow situation, we use B = B0 at all boundaries

Divergence cleanup

Given a computed magnetic field distribution ***B**** from the PDEs above,
we seek a divergence free update to the solution at ***B***$^{n+1}$

$$\nabla \cdot \vec{\boldsymbol{B}}^{\boldsymbol{n}+1} = 0$$

We perform a Hodge-type decomposition of the vector field ***B****

$$\vec{\boldsymbol{B}}^{\boldsymbol{n}+1} = \boldsymbol{B}^{*} - \nabla \psi$$

$$\nabla \cdot \vec{\boldsymbol{B}}^{\boldsymbol{n}+1} = 0 \quad \Rightarrow \nabla^{2} \psi = \nabla \cdot \vec{\boldsymbol{B}}^{*}$$

The Poisson equation above is solved with Dirichlet BC: $\psi = 0$ on all boundaries

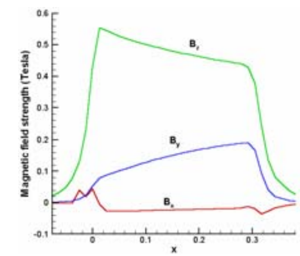
Basic Physics – Major model enhancements

- Control of divergence errors in current density and magnetic field

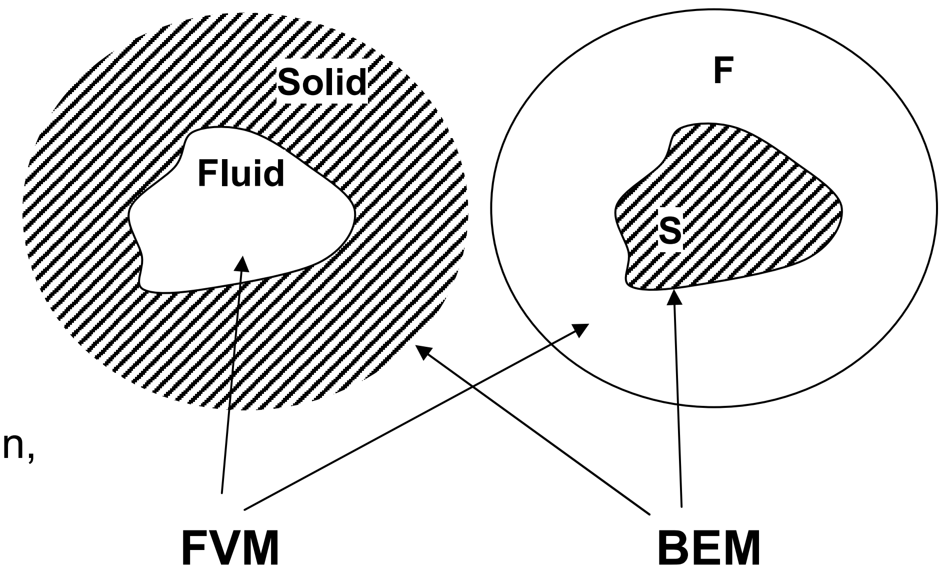
Given a 1-D measurement of B, how best to use it in a 3-D φ code,

ensuring

Div(B) = 0
Curl(B) = 0



- High Ha semi-analytical treatment

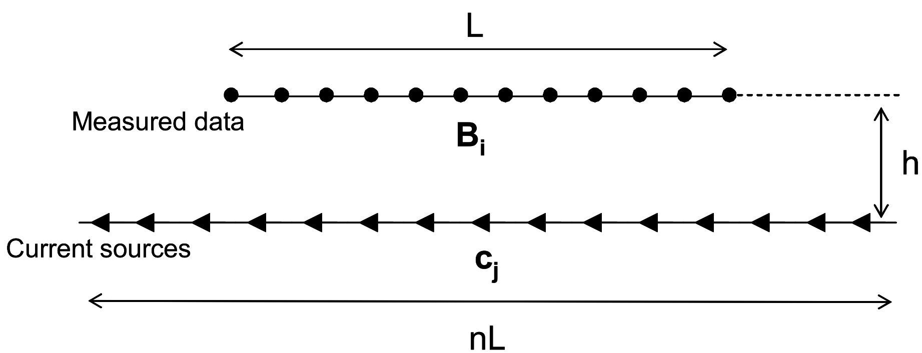


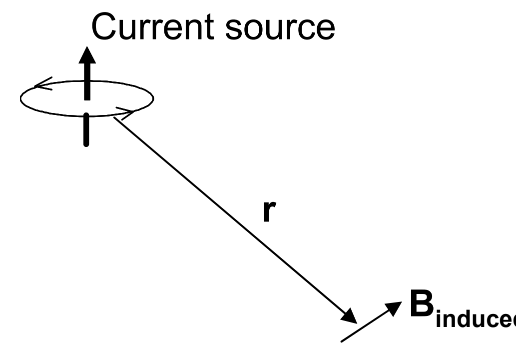
- Boundary element method for B and phi:
  A coupled BEM-FVM strategy as an option,
  could be used as a BC to complement
  thin conducting wall, insulating wall

Reconstruction of 3-D magnetic fields from discrete data

The problem:
Given a set of magnetic field measurements along a line,
to extrapolate it for use in a 3-D code

Procedure:
Locate a set of sources at a distance and compute their strengths
to induce a magnetic field matching measurements





$$\delta\vec{\boldsymbol{B}} = \alpha \frac{\vec{\boldsymbol{r}} \times \delta\vec{\boldsymbol{I}}}{\boldsymbol{r}^3}$$

Biot-Savart law

$$\sum_{j=1}^{N} c_j \delta \vec{B}_{ij} = B_i \quad i = 1 \ldots M$$

$$\underbrace{[A]^T [A]}_{M} \{c_j\} = [A]^T \{B_i\}$$

$$[A]\{c_j\} = \{B_i\} \longrightarrow \{c_j\} = M^{-1}[A]^T \{B_i\}$$

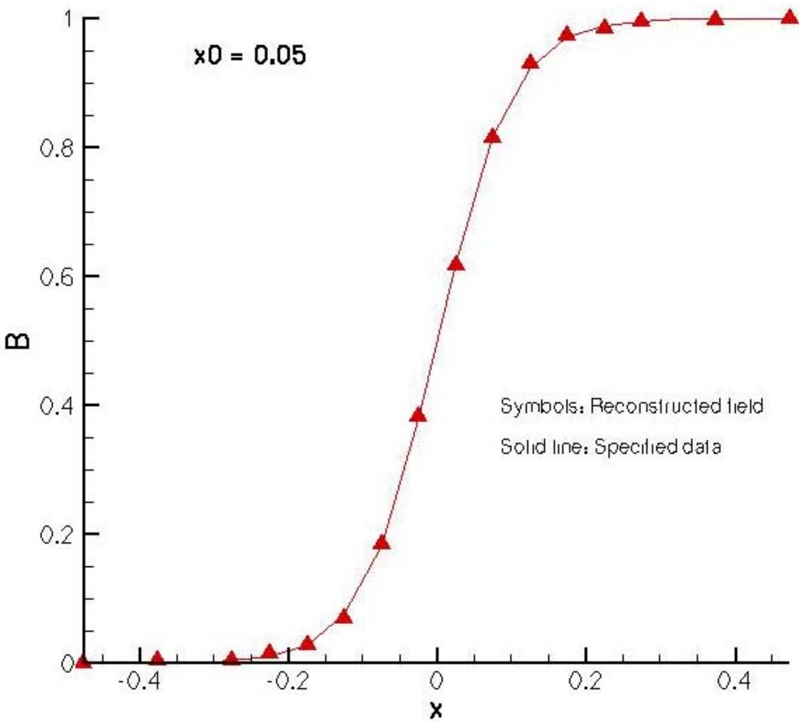
[A] is in general not a square matrix.

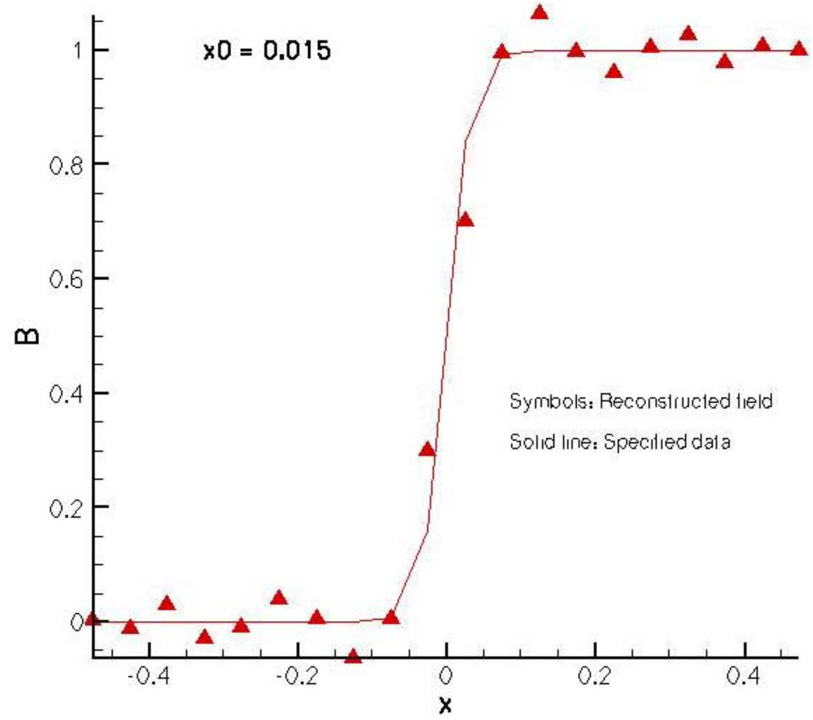
This system can be solved by the method of least squares

Sample application to a field with a gradient (e.g., Sterl – 1990)

Applied field: $$B_y(x) = \frac{1}{1+\exp(-x/x_0)}$$

20 “measurements” matched with 30 sources



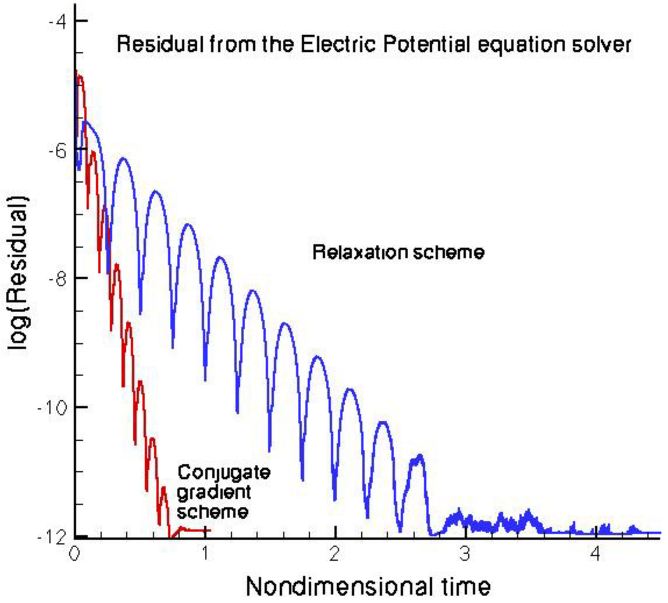


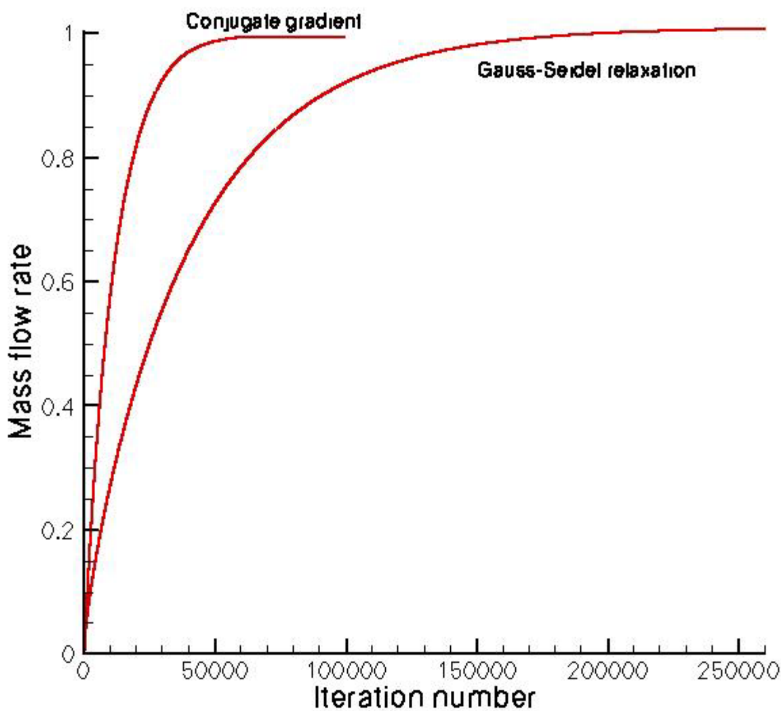
Fast and accurate Poisson solver development

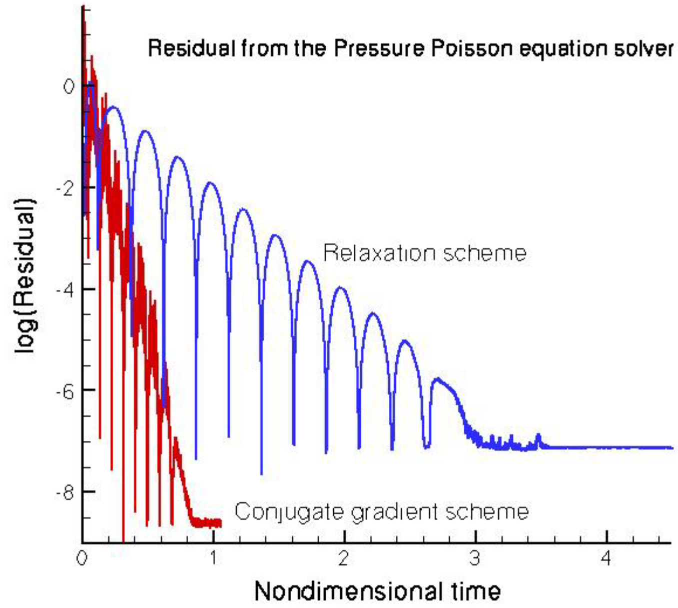
The CG versions of pressure and potential equation solvers are now ready. Gauss-Seidel over-relaxation is available as an option.

Parallelization is complete. div(J) and div(V) seen to be well controlled by switching to CG

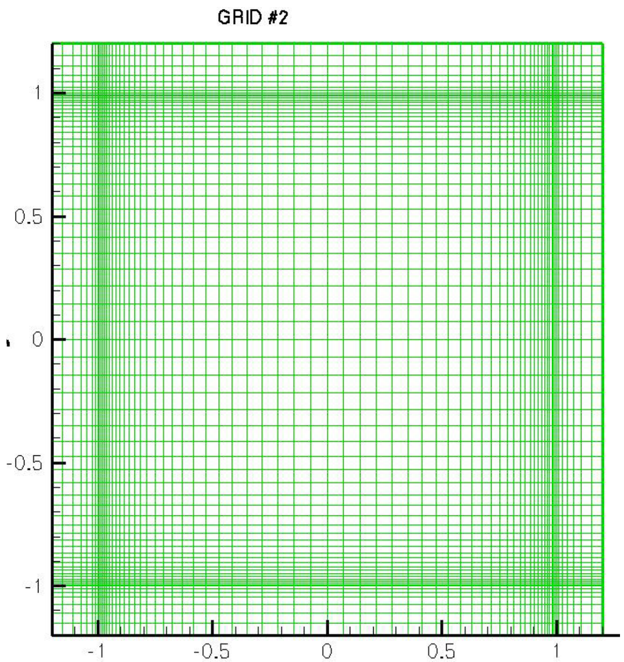
Free surface tests are going on.

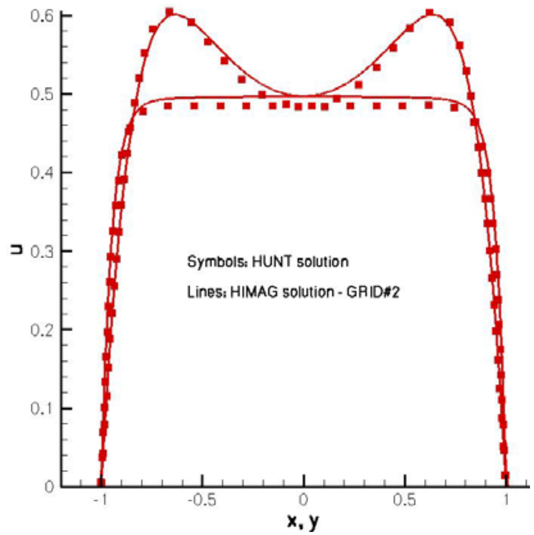


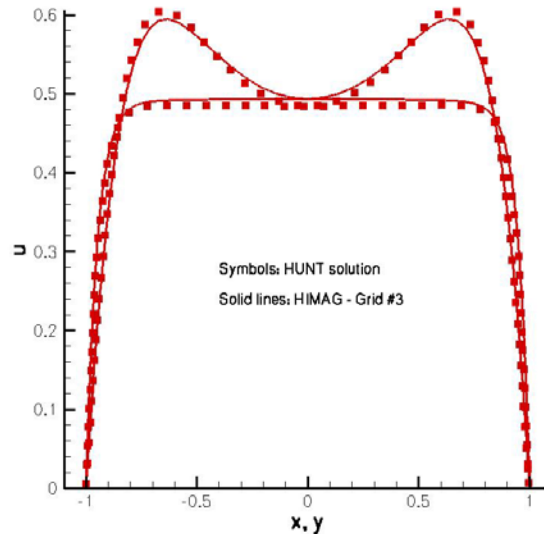


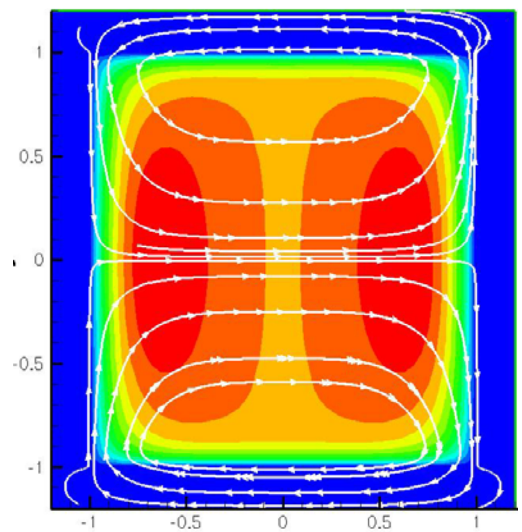


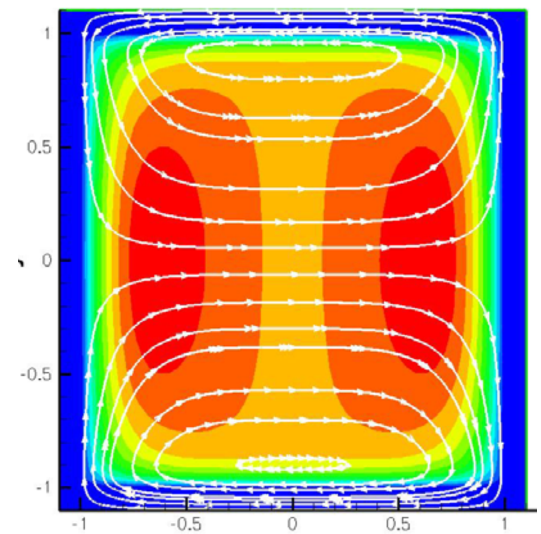
Conducting walls











Ha = 20 with conducting walls, fully developed flow.

Comparison with Hunt's solution.

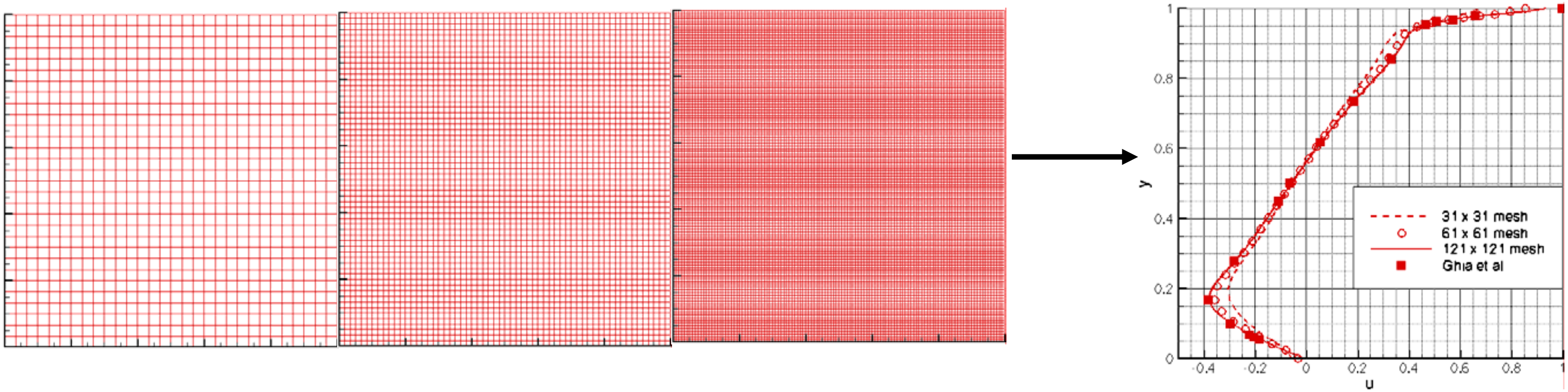
Effect of a wall thickness.

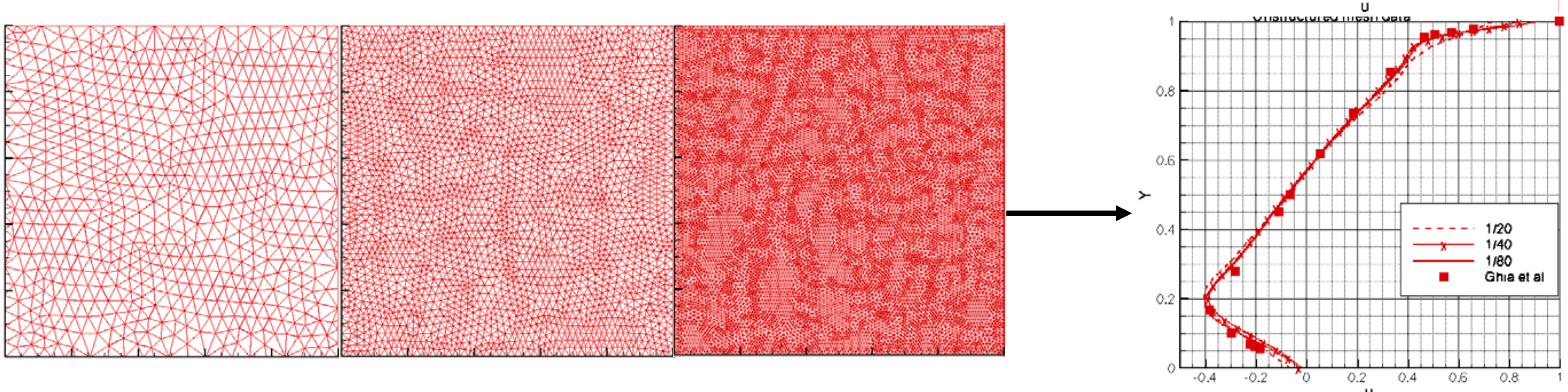
$c_w = 0.1$ (fixed in both cases)

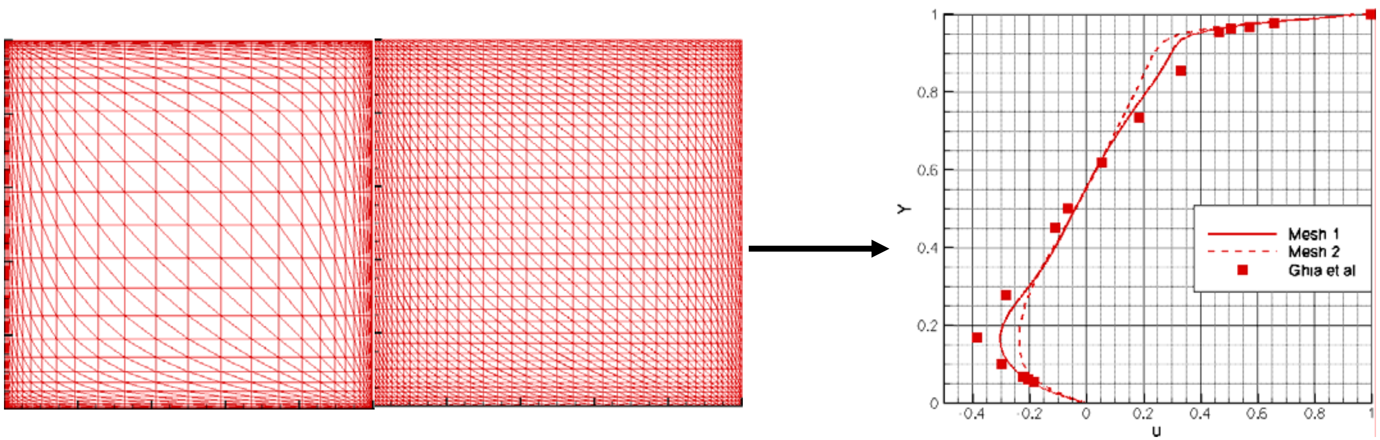
Computational technology and its demonstration:

- Effects of mesh orthogonality, structure
- Order of accuracy of algorithm
- Parallel efficiency
- Applications to sample problems

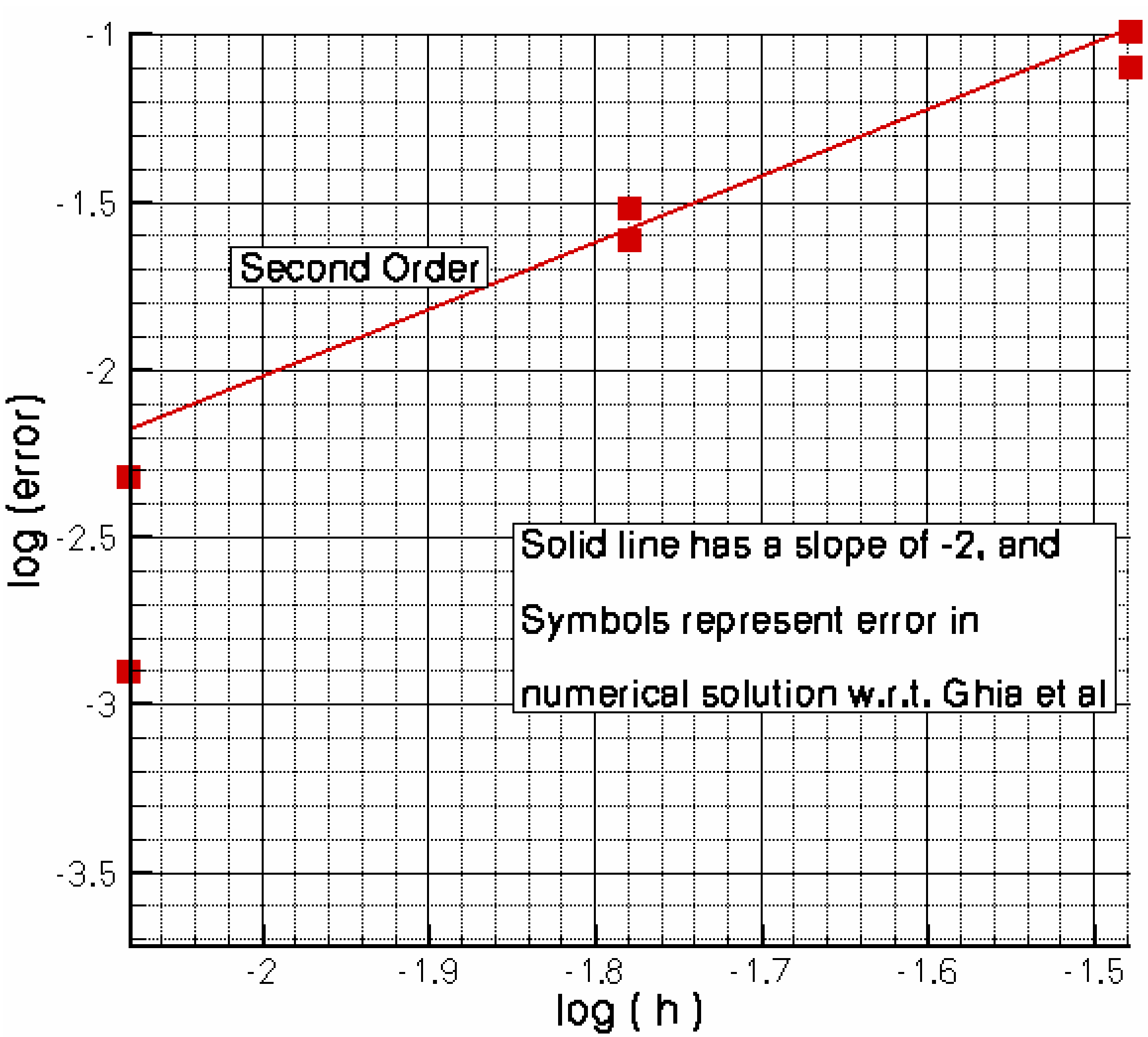
Effect of mesh quality – The driven cavity problem at Re = 1000



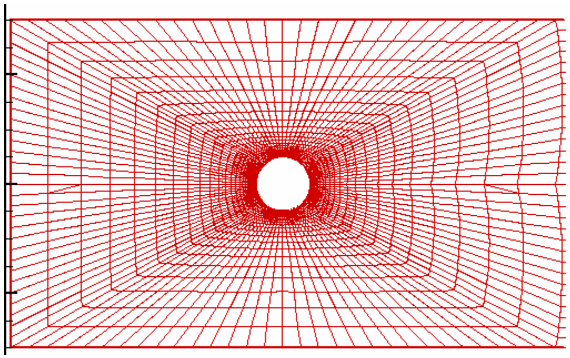




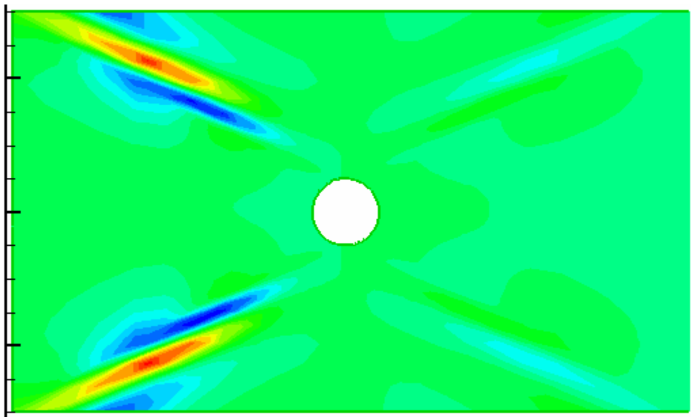
Formal order of accuracy study for the driven cavity problem



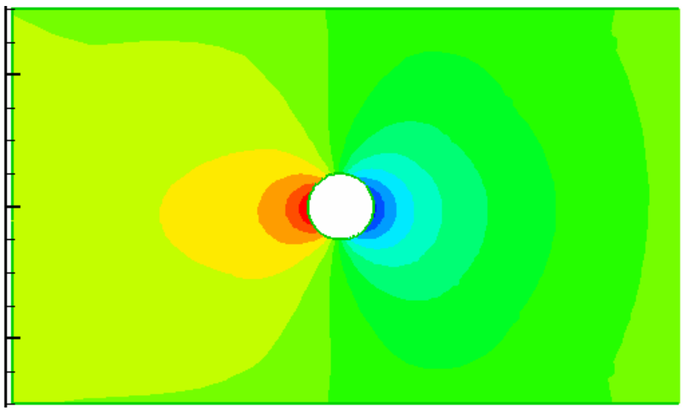
Non-orthogonality corrections imposed



Without mesh-skewness correction

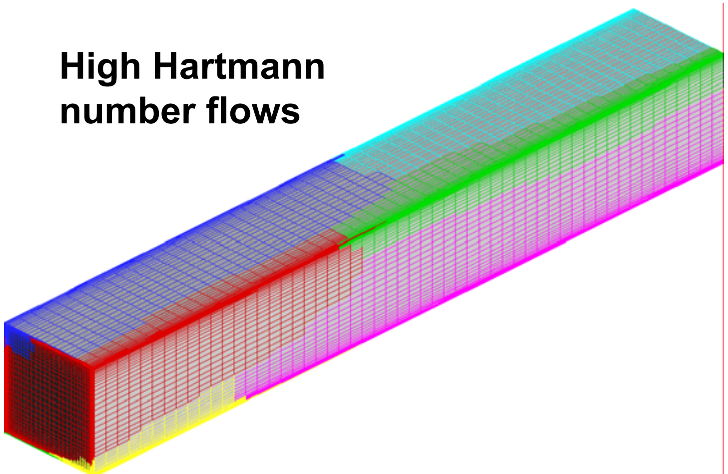


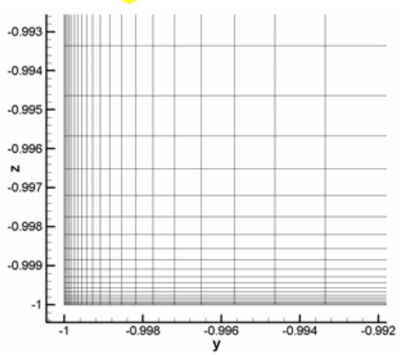
With mesh-skewness correction

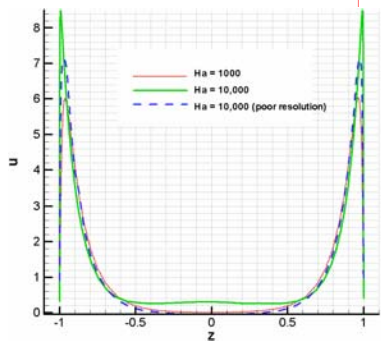


Parallel code execution

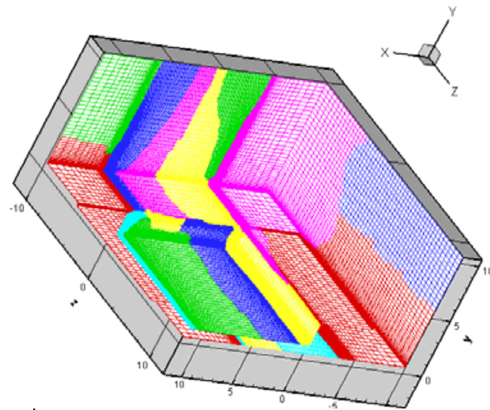
High Hartmann number flows

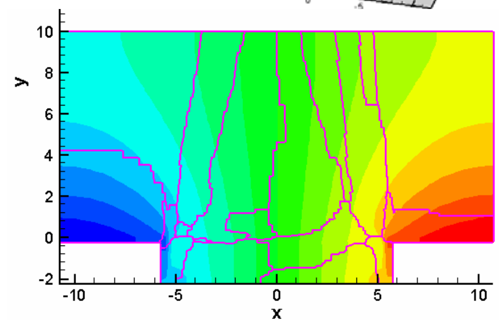


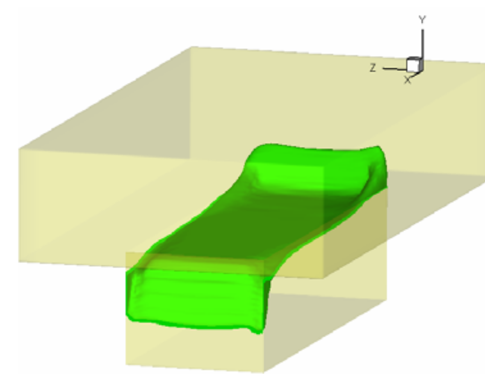




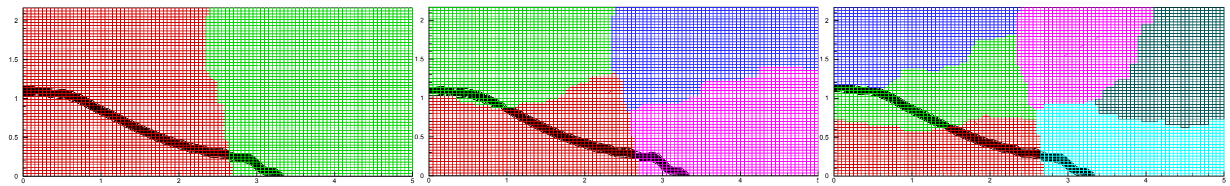
DiMES: MHD sloshing of liquid metal



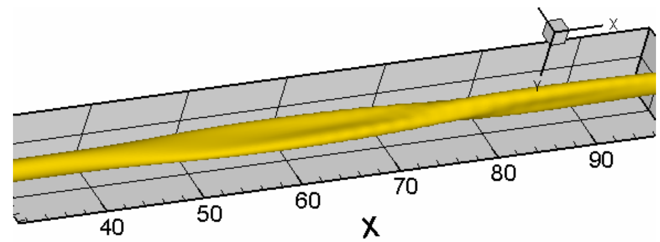




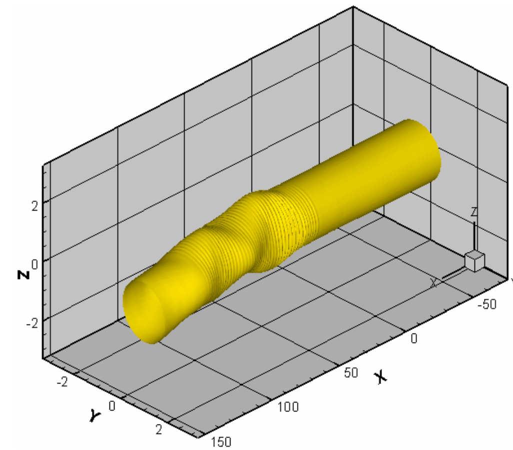
Validation across multi-processors

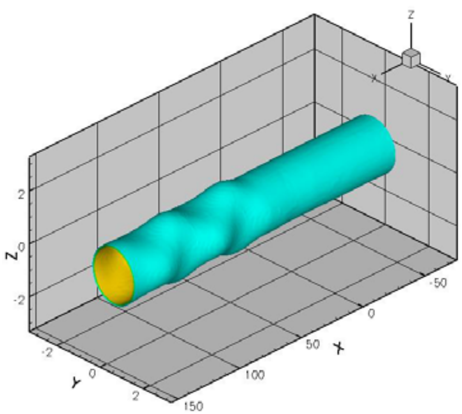


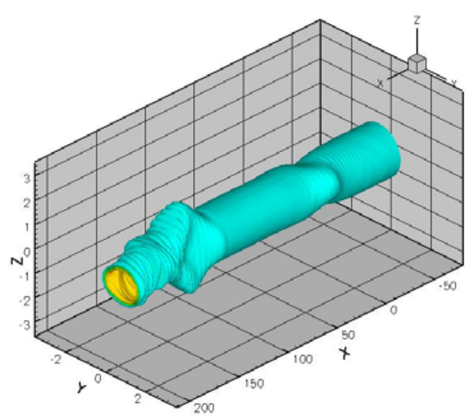
MHD jet flows

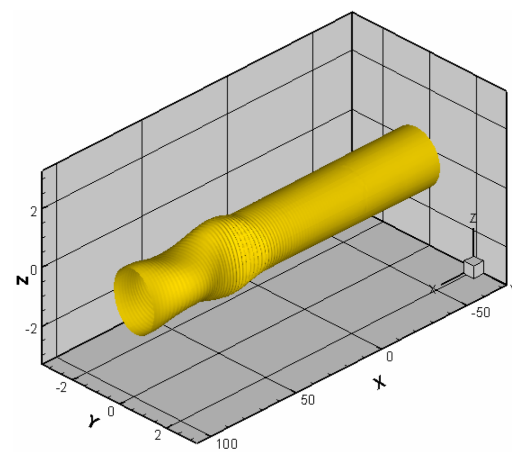


Presently we are trying to study the effect of Reynolds and Hartmann numbers on the modes and stability of a liquid metal jet in a magnetic field with sharp gradients.



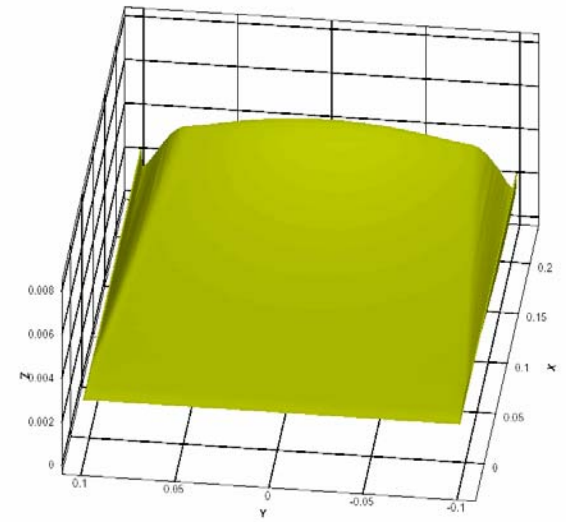


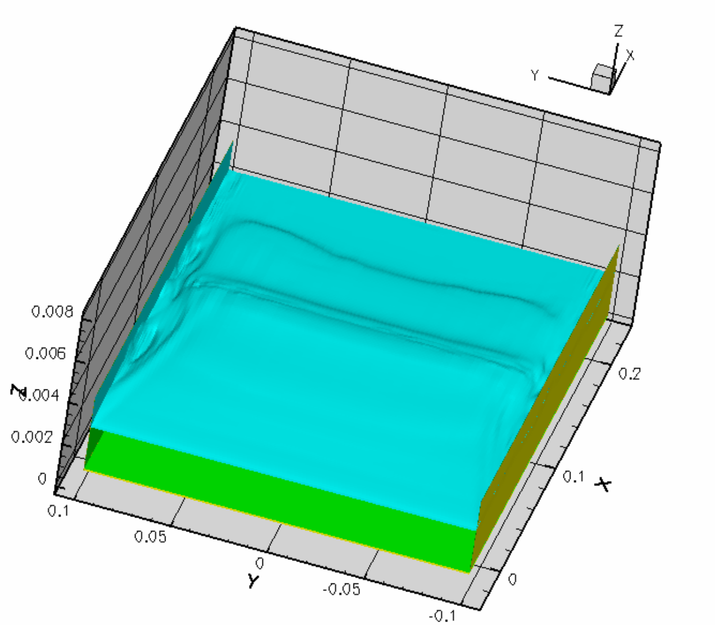


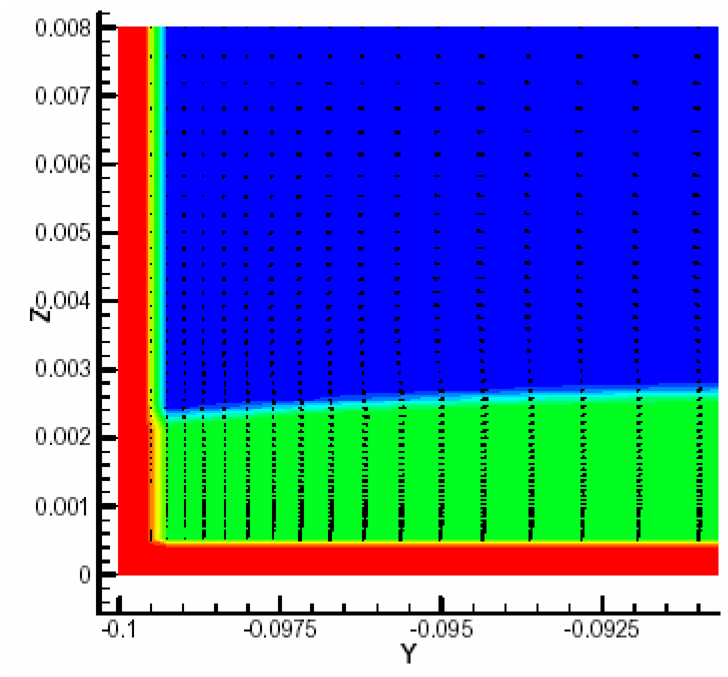


Free surfaces and conducting walls

After a series of troubleshooting exercises, We are now able to run fairly high Re cases of film flows in the presence of conducting walls.







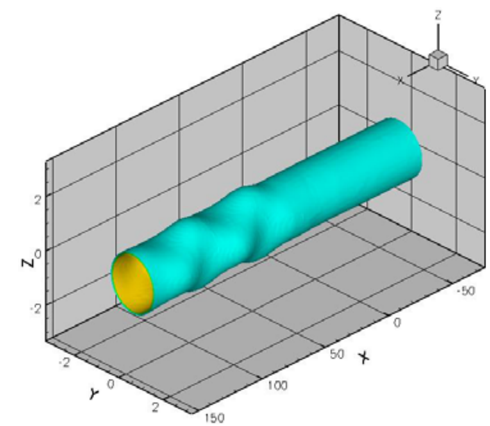
Validation

Methodical validation of the results from MTOR, NSTX-jet flow cases is under way.

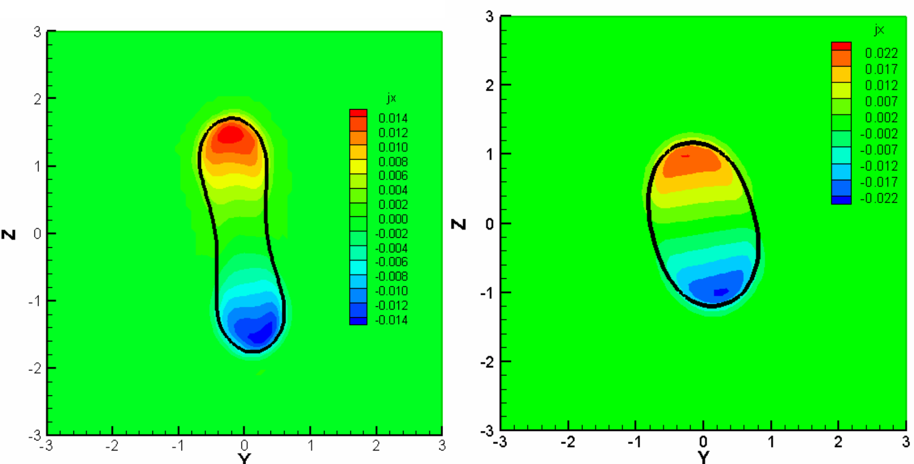
An analytical confirmation of non-MHD flow results will be completed in the coming month.

MHD jet/film flow data from experiments are being Prepared for comparison with HIMAG

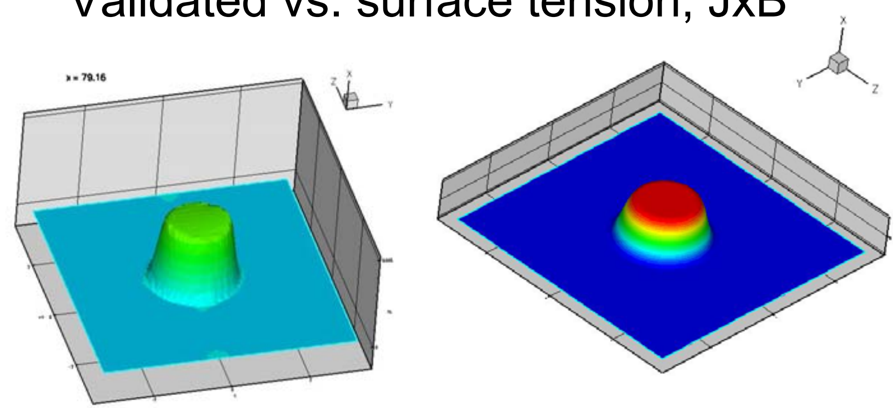
Instability structure for non-MHD cases



Axial current contours



Pressure and density distributions Validated vs. surface tension, JxB



Tasks for the immediate future:

- Completion of the induction equation formulation, ferromagnetic effects
- Development of a general purpose convection-diffusion routine (energy, ke, tritium…)
- Fully implicit execution of free surface flow calculations (right now, we are running in a semi-implicit mode – BCs are explicit.)
- Suggestions...